



Cooperation evolves more when players keep the interaction with unknown players



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ABSTRACT

The existence of cooperation is mysterious. When a cooperator interacts with a cooperator more likely than a defector meets with a cooperator, the evolution of cooperation is possible. Thus far, some such mechanisms (e.g., direct reciprocity and group selection) have been proposed to explain the evolution of cooperation. A recent study considered the case where players can decide to stop the interaction with the current opponent and search for the next opponent, or to continue to interact with the current opponent. In this case, the situation where a cooperator interacts with a cooperator more likely than a defector meets with a cooperator can be realized and the evolution of cooperation is possible. Here, relevant to this mechanism is information deficiency. It is reasonable to suppose that players do not always know what the opponent players did. In this study, we aim to answer the following three questions: Will it promote the evolution of cooperation that the players keep the interaction with unknown players? Besides, will the absence of information about the opponent influence the evolution of cooperation? In addition, it is not obvious which strategy is more likely to evolve, a strategy who hopes to keep the interaction with unknown players or a strategy who stops the interaction with unknown players. By using a mathematical model, we reveal that the evolution of cooperation is more likely when players hope to keep the interaction with unknown players and that information deficiency disturbs the evolution of cooperation and that hoping to keep the interaction with unknown partners is likely to evolve for some cases and stopping the interaction with unknown partners is likely to evolve for other cases.

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1. Introduction

The existence of cooperation can be regarded as mysterious phenomenon since cooperation is costly to the actor [1–3]. In order that cooperation is favored by natural selection, cooperators must have some merit which defectors do not have. When a cooperator interacts with a cooperator more likely than a defector meets with a cooperator, a cooperator is helped more than a defector; hence, cooperation can pay. And some mechanisms under which a cooperator interacts with a cooperator more likely than a defector interacts with a cooperator has been proposed. One mechanism is direct reciprocity [4]. When a direct reciprocator meets a defector, a direct reciprocator withholds future cooperation and turns to defection. Withholding future cooperation and turning to defection can be interpreted as stopping interacting with a defector. And if we interpret it like this, direct reciprocity can be regarded as a mechanism under which a cooperator interacts with a cooperator more likely than a defector interacts with a cooperator. Another mechanism is group selection [5]. Obviously, group selection can be a mechanism under which a cooperator interacts with a cooperator more likely than a defector interacts with a cooperator. Also, structured population enhances the evolution of cooperation by producing the situation in which a cooperator interacts with a cooperator more likely than a defector interacts with a cooperator [6–9].

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When players adopt the strategy which stops the relationship with defectors (called CONCO, Out-for-Tat, MOTH, or Walk Away), cooperators meet cooperators more often than defectors meet cooperators; hence it is considered that cooperative behavior can pay. Actually, previous studies have shown that this mechanism can enable cooperation to evolve [10–28].

However, these studies are based on the assumption that the information about the opponent is always accessible, while it is also reasonable to assume that the information about the opponent will be sometimes absent owing to the imperfect cognitive ability [29–40]. Of course, everyone welcomes and wants to interact with cooperators, while no one is interested in interacting with defectors. However, it is not obvious whether a player welcomes unknown players or not. And it is not obvious if keeping the interaction with unknown players enables cooperation to evolve more. In addition, in such cases where information is sometimes absent, is the evolution of cooperation disturbed? Additionally, it is not obvious which strategy is more likely to evolve, a strategy who hopes to keep the interaction with unknown players or a strategy who stops the interaction with unknown players. Our main goal is to investigate under which case the evolution of cooperation is more likely, when players keep the interaction with unknown players or when players stop the interaction with unknown players, to investigate the effect of the information deficiency on the evolution of cooperation, and to investigate which strategy is more likely to evolve, a strategy who hopes to keep the interaction with unknown players or a strategy who stops the interaction with unknown players.

The paper is organized as: the assumption is given in Section 2; in Section 3, we make analysis; and in Section 4 we summarize our results.

2. Definitions and assumptions

Consider a standard Prisoner's Dilemma (PD) game with payoff matrix $\begin{pmatrix} b-c & -c \\ b & 0 \end{pmatrix}$ with $b > c$, where $b - c$ (or $-c$) is the payoff of a cooperator when it plays against a cooperator (or a defector), and, similarly, b (or 0) is the payoff of a defector when it plays against a cooperator (or a defector) [3,41]. Based on individual's self-interest, we first assume that for both cooperators and defectors, if an individual knows his opponent displaying cooperation (C), then he will continue the interaction with his opponent, and, conversely, if an individual knows his opponent displaying defection (D), he will unilaterally break off the interaction with his opponent [13]. This means that all individuals (including both cooperators and defectors) will respond to defection by merely leaving (i.e. all individuals use OFT) if they can identify their opponents' behavior [18].

Furthermore, we assume that for each of two individuals in a pairwise interaction, (i) the information about his opponent's behavior is somehow blocked with probability e [29–39]; and (ii) if the opponent's behavior cannot be identified, a cooperator will stop the interaction with his opponent with probability β_C , and a cooperator will continue the interaction with complementary probability $1 - \beta_C$ and a defector will stop the interaction with his opponent with probability β_D , and a defector will continue the interaction with complementary probability $1 - \beta_D$. β_C (and β_D) can be regarded as the index of pessimism of cooperators (and defectors). On the other hand, we continue to assume as in the classic repeated PD game that the expected number of rounds between a pair of individuals is limited even if these two individuals would like to continue their interaction [22]. Specifically, we assume that each of interaction pairs will be systematically terminated at any time t with probability ρ , where this probability is independent of individuals' strategies [22]. Finally, we assume that at any time t all single individuals will form new interaction pairs through the random meeting [22].

Based on the above definitions and assumptions, the expected probability that an interaction pair CC will be disbanded at any time t , denoted by ρ_{CC} , can be given by

$$\begin{aligned} \rho_{CC} = & (1 - e)^2 \rho + e^2 \left[\beta_C^2 + 2\beta_C(1 - \beta_C) + (1 - \beta_C)^2 \rho \right] \\ & + 2e(1 - e) \left[\beta_C + (1 - \beta_C) \rho \right]. \end{aligned} \quad (1)$$

Similarly, the expected probability that an interaction pair CD (or DD) will be disbanded at any time t , denoted by ρ_{CD} (or ρ_{DD}) can be given by

$$\begin{aligned} \rho_{CD} = & (1 - e)^2 + e^2 \left[(1 - (1 - \beta_C)(1 - \beta_D)) + (1 - \beta_C)(1 - \beta_D) \rho \right] \\ & + e(1 - e) + e(1 - e) \left[\beta_C + (1 - \beta_C) \rho \right], \\ \rho_{DD} = & (1 - e)^2 + e^2 \left[\beta_D^2 + 2\beta_D(1 - \beta_D) + (1 - \beta_D)^2 \rho \right] \\ & + 2e(1 - e). \end{aligned} \quad (2)$$

Let n_{CC} , n_{CD} and n_{DD} denote the numbers of interaction pairs CC , CD and DD , respectively. Notice that at any time t , the total size of the single-individual group is $n_{CC}\rho_{CC} + n_{CD}\rho_{CD} + n_{DD}\rho_{DD}$, and the frequencies of C and D in the single-individual

group are

$$\frac{2n_{CC}\rho_{CC} + n_{CD}\rho_{CD}}{2(n_{CC}\rho_{CC} + n_{CD}\rho_{CD} + n_{DD}\rho_{DD})},$$

$$\frac{2n_{DD}\rho_{CC} + n_{CD}\rho_{CD}}{2(n_{CC}\rho_{CC} + n_{CD}\rho_{CD} + n_{DD}\rho_{DD})},$$

respectively. Then, the changes of n_{CC} , n_{CD} and n_{DD} can be described as

$$\begin{aligned} \frac{dn_{CC}}{dt} &= -n_{CC}\rho_{CC} + \frac{(2n_{CC}\rho_{CC} + n_{CD}\rho_{CD})^2}{4(n_{CC}\rho_{CC} + n_{CD}\rho_{CD} + n_{DD}\rho_{DD})}, \\ \frac{dn_{CD}}{dt} &= -n_{CD}\rho_{CD} + \frac{2(2n_{CC}\rho_{CC} + n_{CD}\rho_{CD})(2n_{DD}\rho_{DD} + n_{CD}\rho_{CD})}{4(n_{CC}\rho_{CC} + n_{CD}\rho_{CD} + n_{DD}\rho_{DD})}, \\ \frac{dn_{DD}}{dt} &= -n_{DD}\rho_{DD} + \frac{(2n_{DD}\rho_{DD} + n_{CD}\rho_{CD})^2}{4(n_{CC}\rho_{CC} + n_{CD}\rho_{CD} + n_{DD}\rho_{DD})}. \end{aligned} \tag{3}$$

From $dn_{CC}/dt = 0$, $dn_{CD}/dt = 0$ and $dn_{DD}/dt = 0$, we have

$$(n_{CD}\rho_{CD})^2 = 4n_{CC}n_{DD}\rho_{CC}\rho_{DD}. \tag{4}$$

Let P_{CC} , P_{CD} and P_{DD} denote the proportions of interaction pairs CC , CD and DD , respectively, where the frequency of C , denoted by x , is $x = P_{CC} + P_{CD}/2$ and the frequency of D is $1 - x = P_{DD} + P_{CD}/2$. Then, Eq. (4) can be rewritten as

$$(P_{CD}\rho_{CD})^2 = 4P_{CC}P_{DD}\rho_{CC}\rho_{DD} \tag{5}$$

[22]. It is easy to see that P_{CD} can be solved as:

$$P_{CD} = 2x(1 - x) \tag{6}$$

if $\rho_{CD}^2 = \rho_{CC}\rho_{DD}$,

$$P_{CD} = -\frac{\rho_{CC}\rho_{DD}}{\rho_{CD}^2 - \rho_{CC}\rho_{DD}} + \sqrt{\left(\frac{\rho_{CC}\rho_{DD}}{\rho_{CD}^2 - \rho_{CC}\rho_{DD}}\right)^2 + \frac{4x(1 - x)\rho_{CC}\rho_{DD}}{\rho_{CD}^2 - \rho_{CC}\rho_{DD}}} \tag{7}$$

if $\rho_{CD}^2 > \rho_{CC}\rho_{DD}$, and

$$P_{CD} = \frac{\rho_{CC}\rho_{DD}}{\rho_{CC}\rho_{DD} - \rho_{CD}^2} - \sqrt{\left(\frac{\rho_{CC}\rho_{DD}}{\rho_{CC}\rho_{DD} - \rho_{CD}^2}\right)^2 - \frac{4x(1 - x)\rho_{CC}\rho_{DD}}{\rho_{CC}\rho_{DD} - \rho_{CD}^2}} \tag{8}$$

if $\rho_{CD}^2 < \rho_{CC}\rho_{DD}$.

Notice that compared to the change of x , the changes of P_{CC} , P_{CD} and P_{DD} should be considered to be fast variables [22]. So, in this study, we define that Eq. (5) holds at any time t .

3. Model and analysis

Let $\phi_{C|C}$ (or $\phi_{D|C} = 1 - \phi_{C|C}$) denote the probability that a C -individual has an opponent displaying C (or D) at time infinitely large t , and, similarly, $\phi_{C|D}$ (or $\phi_{D|D} = 1 - \phi_{C|D}$) the probability that a D -individual has an opponent displaying C (or D). Then, we have

$$\begin{aligned} \phi_{C|C} &= \frac{2P_{CC}}{2P_{CC} + P_{CD}} = \frac{2x - P_{CD}}{2x}, \\ \phi_{C|D} &= \frac{P_{CD}}{2P_{DD} + P_{CD}} = \frac{P_{CD}}{2(1 - x)}. \end{aligned} \tag{9}$$

Based on the definitions of $\phi_{C|C}$ and $\phi_{C|D}$, the expected payoffs of C and D , denoted by π_C and π_D , respectively, can be given by

$$\begin{aligned} \pi_C &= \phi_{C|C}(b - c) - (1 - \phi_{C|C})c \\ &= \phi_{C|C} b - c = \frac{2x - P_{CD}}{2x} b - c, \\ \pi_D &= \phi_{C|D} b = \frac{P_{CD}}{2(1 - x)} b. \end{aligned} \tag{10}$$

Notice that

$$\begin{aligned} \pi_C - \pi_D &= (\phi_{C|C} - \phi_{C|D})b - c \\ &= \left(1 - \frac{P_{CD}}{2x(1-x)}\right)b - c. \end{aligned} \tag{11}$$

Obviously, if it is assumed that the population size is large enough, then the time evolution of x obeys a simple differential equation $dx/dt = x(1-x)(\pi_C - \pi_D) = x(1-x)(b-c) - bP_{CD}/2$ where P_{CD} is assumed to be at the temporal equilibrium.

So, it is easy to see that $\pi_C > \pi_D$ if $\phi_{C|C} - \phi_{C|D} > c/b$. Specifically, if $\rho_{CC}\rho_{DD} \geq \rho_{CD}^2$, then we must have $\pi_C < \pi_D$. On the other hand, for the situation with $\rho_{CC}\rho_{DD} < \rho_{CD}^2$, it is also easy to see that $\pi_C > \pi_D$ if

$$x(1-x)\left(1 - \frac{c}{b}\right)^2 - \frac{c}{b} \cdot \frac{\rho_{CC}\rho_{DD}}{\rho_{CD}^2 - \rho_{CC}\rho_{DD}} > 0. \tag{12}$$

For convenience, let $\nu = \rho_{CC}\rho_{DD} / \rho_{CD}^2$. We can see that: (i) if $\nu > (1-c/b)^2 / (1+c/b)^2$, then we have $\pi_C < \pi_D$; and (ii) if $\nu < (1-c/b)^2 / (1+c/b)^2$, then we have

$$x_{1,2}^* = \frac{1}{2} \pm \frac{1}{2} \sqrt{1 - \frac{4c/b}{(1-c/b)^2} \cdot \frac{\nu}{1-\nu}} \tag{13}$$

with $x_2^* < x_1^*$ such that $\pi_C < \pi_D$ when x is in the interval $0 < x < x_2^*$ or $x_1^* < x < 1$, and $\pi_C > \pi_D$ when x is in the interval $x_2^* < x < x_1^*$ (see also [22]).

Therefore, when the frequency of cooperators is 0 or 1, the payoff of defectors is larger than that of cooperators; thus, the population of cooperators is always unstable against the invasion by a defector (i.e., the initial evolution of cooperation is impossible) and the population of defectors is always stable against the invasion by a cooperator (i.e., the maintenance of cooperation is impossible). Eq. (13) shows that as ν becomes smaller, the internal unstable equilibrium decreases and the internal stable equilibrium increases. Therefore, as ν becomes smaller, the basin of attraction for cooperators becomes larger and the evolution of cooperation is more likely.

From Eqs. (1) and (2), ν can be expressed as

$$\nu(\beta_C, \beta_D) = \frac{[(1-e)^2\rho + e^2\mu_1(\beta_C, \beta_D) + 2e(1-e)\mu_2(\beta_C, \beta_D)][(1-e^2) + e^2\mu_3(\beta_C, \beta_D)]}{[(1-e) + e^2\mu_4(\beta_C, \beta_D) + e(1-e)\mu_2(\beta_C, \beta_D)]^2}, \tag{14}$$

where

$$\begin{aligned} \mu_1(\beta_C, \beta_D) &= \beta_C(2 - \beta_C) + (1 - \beta_C)^2\rho, \\ \mu_2(\beta_C, \beta_D) &= \beta_C + (1 - \beta_C)\rho, \\ \mu_3(\beta_C, \beta_D) &= \beta_D(2 - \beta_D) + (1 - \beta_D)^2\rho, \\ \mu_4(\beta_C, \beta_D) &= (\beta_C + \beta_D - \beta_C\beta_D) + (1 - \beta_C)(1 - \beta_D)\rho. \end{aligned} \tag{15}$$

3.1. Case where $\beta_C = \beta_D$

In this subsection, we examine under which case the evolution of cooperation is more encouraged, when players are optimistic or when players are pessimistic. For simplicity, we assume $\beta_C = \beta_D = \beta$. Substituting $\beta_C = \beta_D = \beta$ into Eq. (14), we obtain

$$\nu(\beta, \beta) = \frac{[(1-e)^2\rho + e^2\mu_5(\beta) + 2e(1-e)\mu_6(\beta)][(1-e^2) + e^2\mu_5(\beta)]}{[(1-e) + e^2\mu_5(\beta) + e(1-e)\mu_6(\beta)]^2}, \tag{16}$$

where

$$\begin{aligned} \mu_5(\beta) &= \beta(2 - \beta) + (1 - \beta)^2\rho, \\ \mu_6(\beta) &= \beta + (1 - \beta)\rho. \end{aligned} \tag{17}$$

By using Eqs. (13) and (16), the relationship between β and the internal stable (unstable) equilibrium is illustrated (see Fig. 1(a)(b)). We can see that as β increases, the basin of attraction for cooperators shrinks (see Fig. 1(a)(b)), and but when b/c is large enough, even when $\beta = 1$, defectors do not dominate cooperators (see Fig. 1(b)).

Then we have

$$\frac{d\nu(\beta, \beta)}{d\beta} = \frac{2(1-e)^2e(1+e(1-2\beta))(1-\rho)^2}{[1-e(1-\beta)(1-e\beta)(1-\rho)]^3}. \tag{18}$$

It is easy to see that the right-hand side of Eq. (18) is positive. This means that the evolution of cooperation is more likely when players keep the interaction with a higher probability when the partner's behavior is unknown.

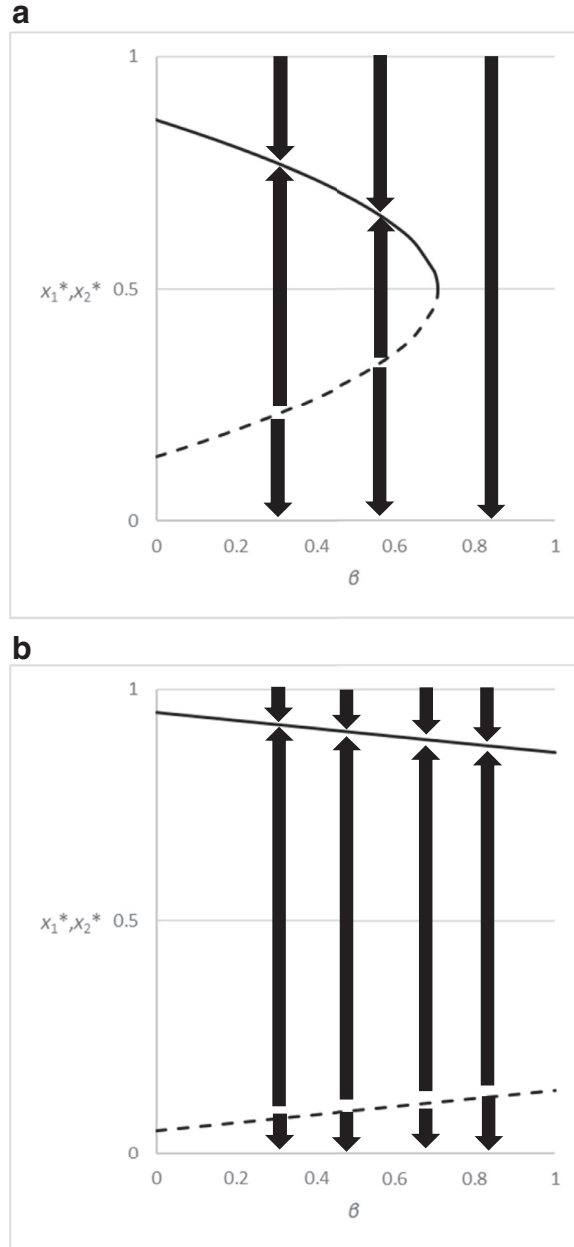


Fig. 1. A bifurcation diagram: how β influences the internal stable equilibrium and the internal unstable equilibrium. Fig. 1 illustrates the relationship between β and the internal stable (unstable) equilibrium, with β indexed on the horizontal axis and the frequency of cooperators shown on the vertical axis. The solid line indicates the internal stable equilibrium, and the broken line indicates the internal unstable equilibrium. Fig. 1 indicates that as β increases, the basin of attraction for cooperators shrinks. Besides, by comparing Fig. 1(a) with Fig. 1(b), it turns out that when b/c is larger, the evolution of cooperation is more likely. The parameter values used are $\rho = 0.2$ and $e = 0.2$ in Fig. 1(a)(b). The parameters values used are $b/c = 5$ and $b/c = 10$ in Fig. 1(a) and in Fig. 1(b), respectively.

And we also have

$$\frac{dv(\beta, \beta)}{de} = \frac{2(1-e)(1-\rho)(\beta + (1-\beta)(e\beta(2-e)(1-\rho) + \rho))}{[1 - e(1-\beta)(1-e\beta)(1-\rho)]^3}. \tag{19}$$

Obviously, the right-hand side of Eq. (19) is positive. This means that the evolution of cooperation is more likely when information deficiency is less.

From Eqs. (16) and (17), we can obtain Fig. 2. In Fig. 2, we show the effect of β and e on the evolution of cooperation. Defectors dominate cooperators if and only if the value of β is greater than the value on the blue solid line and the red

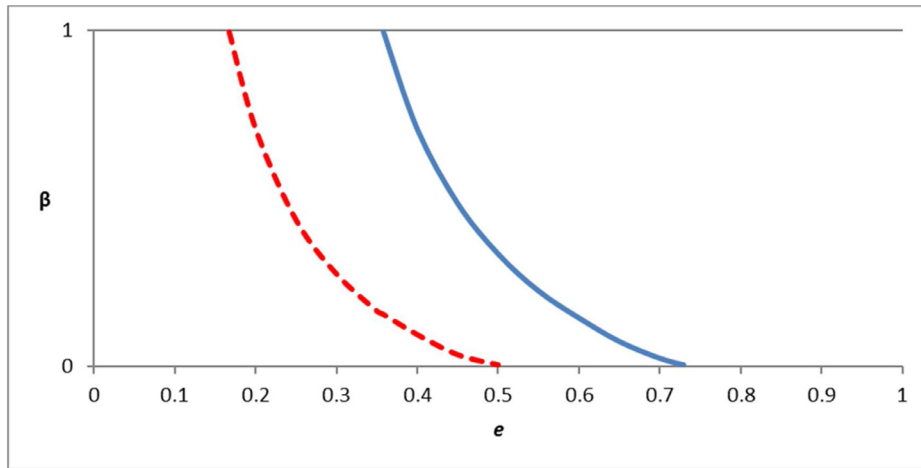


Fig. 2. Effect of β and e on the evolution of cooperation. Fig. 2 illustrates the relationship between e and β , with e indexed on the horizontal axis and β shown on the vertical axis. The combination of e and β is above the line, defectors dominate cooperators. As e decreases, the evolution of cooperation is more likely. As β decreases, the evolution of cooperation is more likely. As b/c increases, the evolution of cooperation is more likely. The parameters used are $\rho = 0.2$, $b/c = 10$ (blue solid line) and $b/c = 5$ (red dotted line). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

dotted line. On the other hand, the system has both a stable internal equilibrium and a unstable internal equilibrium if and only if the value of β is smaller than the value on the blue solid line and the red dotted line (Fig. 2). Fig. 2 shows that the evolution of cooperation is more likely when players keep the interaction with an unknown opponent and that cooperation is more likely to evolve when the information deficiency is less.

3.2. Case where β_C is not necessarily equal to β_D

In the previous Section (3.1), we examined the effect of β on the evolution of cooperation. In this Section (3.2), we examine the evolution of β_C (β_D) itself (i.e., which is more likely to evolve, a strategy who hopes to keep the interaction with unknown players or a strategy who stops the interaction with unknown players) by examining the case where β_C is not necessarily equal to β_D .

Firstly, we examine the case where cooperators stop the interaction with unknown players. Substituting $\beta_C = 1$ into Eq. (14), we have

$$v(1, \beta_D) = (1 - e^2(1 - \beta_D)^2(1 - \rho))(1 - (1 - \rho)(1 - e)^2). \quad (20)$$

Therefore, we get

$$\frac{dv(1, \beta_D)}{d\beta_D} = 2e^2(1 - \beta_D)(1 - \rho)(1 - (1 - \rho)(1 - e)^2) > 0. \quad (21)$$

Hence, $v(1, \beta_D)$ increases monotonically with increasing β_D . Therefore, it turns out that against a pessimistic cooperator ($\beta_C = 1$), a pessimistic defector (large β_D) is more likely to evolve than an optimistic defector (small β_D).

Secondly, we examine the case where cooperators hope to keep the interaction with unknown players. Substituting $\beta_C = 0$ into Eq. (14), we have

$$v(0, \beta_D) = \frac{(1 - e^2(1 - \beta_D)^2(1 - \rho))\rho}{[1 - e(1 - \rho) + e^2\beta_D(1 - \rho)]^2}. \quad (22)$$

Therefore, we get

$$\frac{dv(0, \beta_D)}{d\beta_D} = -\frac{2e^2\rho(1 - \rho)(\beta_D + e(1 - e)(1 - \rho)(1 - \beta_D))}{[1 - e(1 - e\beta_D)(1 - \rho)]^3} < 0. \quad (23)$$

Hence, $v(0, \beta_D)$ decreases monotonically with increasing β_D . Therefore, it turns out that against an optimistic cooperator ($\beta_C = 0$), an optimistic defector (small β_D) is more likely to evolve than a pessimistic defector (large β_D).

Thirdly, we examine the case where defectors stop the interaction with unknown players. Substituting $\beta_D = 1$ into Eq. (14), we have

$$v(\beta_C, 1) = \frac{e\beta_C(2 - e\beta_C)(1 - \rho) + \rho}{[1 - e(1 - e)(1 - \rho)(1 - \beta_C)]^2}. \quad (24)$$

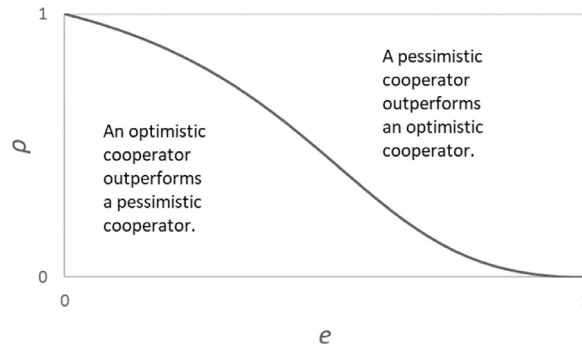


Fig. 3. Against an optimistic defector, which is more likely to evolve, an optimistic cooperator or a pessimistic cooperator? Fig. 3 illustrates the relationship between e and ρ , with e indexed on the horizontal axis and ρ shown on the vertical axis. The critical ρ (ρ^*) decreases with increasing e . When the combination of e and ρ is above this line, a pessimistic cooperator is more likely to evolve than an optimistic cooperator against an optimistic defector.

Therefore, we get

$$\frac{dv(\beta_C, 1)}{d\beta_C} = \frac{2e(1-\rho)((1-e)^2(1-\rho)(1-e\beta_C) + e(1-\beta_C))}{[(1-e(1-e)(1-\rho)(1-\beta_C))^3]} > 0. \tag{25}$$

Hence, $v(\beta_C, 1)$ increases monotonically with increasing β_C . Therefore, it turns out that against a pessimistic defector ($\beta_D = 1$), an optimistic cooperator (small β_C) is more likely to evolve than a pessimistic cooperator (large β_C).

Fourthly, we examine the case where defectors hope to keep the interaction with unknown players. Substituting $\beta_D = 0$ into Eq. (14), we have

$$v(\beta_C, 0) = \frac{(1-e^2(1-\rho))(e\beta_C(2-e\beta_C)(1-\rho) + \rho)}{[1-e(1-\rho)(1-\beta_C)]^2}. \tag{26}$$

Here, by using (26), we get

$$v(0, 0) - v(1, 0) = (\rho - \rho^*)(\rho - \rho^{**}) \frac{e^2(1-e)^2(1-\rho)(1-e^2(1-\rho))}{[1-e(1-\rho)]^2}, \tag{27}$$

where

$$\begin{aligned} \rho^* &= \frac{-2 + 5e - 6e^2 + 2e^3 + \sqrt{4 - 12e + 13e^2 - 4e^3}}{2e(1-e)^2}, \\ \rho^{**} &= \frac{-2 + 5e - 6e^2 + 2e^3 - \sqrt{4 - 12e + 13e^2 - 4e^3}}{2e(1-e)^2}. \end{aligned} \tag{28}$$

Hence, if and only if $\rho > \rho^*$ holds true, $v(0, 0) > v(1, 0)$. In Fig. 3, we know that the critical ρ (ρ^*) decreases as e increases. Therefore, it turns out that against an optimistic defector ($\beta_D = 0$), when e (the information deficiency) is larger than a threshold, a pessimistic cooperator ($\beta_C = 1$) is more likely to evolve than an optimistic cooperator ($\beta_C = 0$), and when e is smaller than the threshold, an optimistic cooperator ($\beta_C = 0$) is more likely to evolve than a pessimistic cooperator ($\beta_C = 1$).

4. Discussion

In natural selection, cooperation is always considered to be conditional. As one type of the conditional cooperation, the mechanism that players can choose to keep the interaction or to stop the interaction is introduced by Zhang et al. [21] and Zheng et al. [22].

If the information about the opponent is absent, it is not sure what to do will promote the evolution of cooperation between keeping or stopping the interaction with an unknown opponent. In addition, it is not obvious how the deficiency about the opponent information affects the evolution of cooperation. In Section 3.1, we tackle on these two questions. By studying the conditional repeated C-D game (i.e., cooperator and defector) with the same probability for both cooperators and defectors to continue the interaction with unknown players, it turns out that when players hope to keep the interaction with an unknown opponent, cooperation is more likely to evolve. And it also turns out that the information deficiency disturbs the evolution of cooperation.

Moreover, it is not obvious which strategy is more likely to evolve, a strategy who hopes to keep the interaction with unknown players or a strategy who stops the interaction with unknown players. In Section 3.2, we tackle on this question. By studying the case with two independent probabilities for cooperators and defectors to continue the interaction with unknown players, we revealed the following four results: (i) Against a pessimistic cooperator, a pessimistic defector is more likely to evolve than an optimistic defector. (ii) Against an optimistic cooperator, an optimistic defector is more likely to evolve than a pessimistic defector. (iii) Against a pessimistic defector, an optimistic cooperator is more likely to evolve

than a pessimistic cooperator. (iv) Against an optimistic defector, an optimistic cooperator is more likely to evolve than a pessimistic cooperator when the information deficiency is less than the threshold, and a pessimistic cooperator is more likely to evolve than an optimistic cooperator when the information deficiency is more than the threshold.

A previous theoretical study has revealed that cooperation is less likely to evolve as information about the opponent's behavior becomes less in repeated games WITHOUT an "opt out" option [30]. Our present theoretical study has revealed that this result holds true also in repeated games WITH an "opt out" option. Thus, the previous theoretical study and this present theoretical study have shown that information deficiency has a negative impact on the evolution of cooperation, irrespective of the presence or absence of an "opt out" option. What did previous experimental studies show about the effect of information imperfection on cooperation? An experimental study has shown that cooperation is less observed as information about the opponent's behavior becomes less in repeated games WITHOUT an "opt out" option [40]. This observation is consistent with the previous theoretical study [30]. On the other hand, to the best of our knowledge, we are not aware of any experimental studies which examined how cooperation is affected by information deficiency in repeated games WITH an "opt out" option. Any experimental studies which could show that as information is less, cooperation is less observed in repeated games WITH an "opt out" option could be seen as an interesting confirmation of this present theoretical study. Further study on this issue is required.

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